

where m and b are constants. When $h = 0$, we are given that $T = 20$, so

$$20 = m(0) + b$$

$$b = 20$$

Thus, we have

$$T = mh + 20$$

When $h = 1$, we have $T = 10$ and so

$$10 = m(1) + 20$$

$$m = 10 - 20 = -10$$

The required expression is

$$T = -10h + 20$$

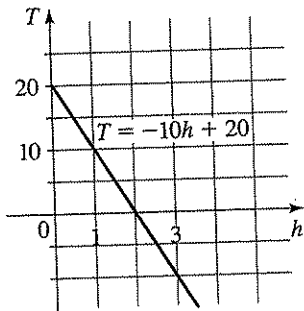


Figure 19

(b) The graph is sketched in Figure 19. The slope is $m = -10^\circ\text{C}/\text{km}$, and this represents the rate of change of temperature with respect to distance above the ground. So the temperature *decreases* 10°C per kilometer of height.

(c) At a height of $h = 2.5$ km, the temperature is

$$T = -10(2.5) + 20 = -25 + 20 = -5^\circ\text{C}$$

1.10 Exercises

1–8 ■ Find the slope of the line through P and Q .

1. $P(0, 0), Q(4, 2)$

2. $P(0, 0), Q(2, -6)$

3. $P(2, 2), Q(-10, 0)$

4. $P(1, 2), Q(3, 3)$

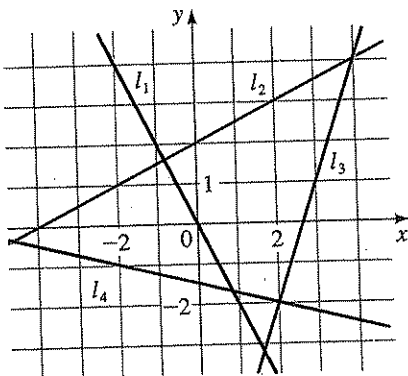
5. $P(2, 4), Q(4, 3)$

6. $P(2, -5), Q(-4, 3)$

7. $P(1, -3), Q(-1, 6)$

8. $P(-1, -4), Q(6, 0)$

9. Find the slopes of the lines l_1, l_2, l_3 , and l_4 in the figure below.

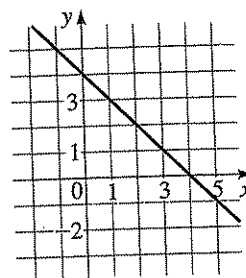


10. (a) Sketch lines through $(0, 0)$ with slopes $1, 0, \frac{1}{2}, 2$, and -1 .

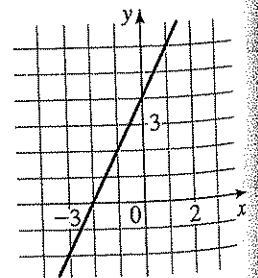
(b) Sketch lines through $(0, 0)$ with slopes $\frac{1}{3}, \frac{1}{2}, -\frac{1}{3}$, and 3 .

11–14 ■ Find an equation for the line whose graph is sketched.

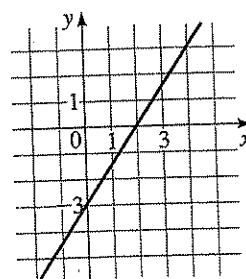
11.



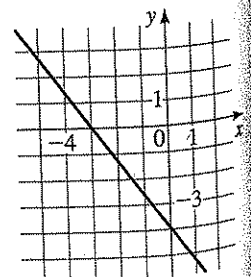
12.



13.



14.



15-34 ■ Find an equation of the line that satisfies the given conditions.

15. Through (2, 3); slope 1
16. Through (-2, 4); slope -1
17. Through (1, 7); slope $\frac{2}{3}$
18. Through (-3, -5); slope $-\frac{7}{2}$
19. Through (2, 1) and (1, 6)
20. Through (-1, -2) and (4, 3)
21. Slope 3; y-intercept -2
22. Slope $\frac{2}{3}$; y-intercept 4
23. x-intercept 1; y-intercept -3
24. x-intercept -8; y-intercept 6
25. Through (4, 5); parallel to the x-axis
26. Through (4, 5); parallel to the y-axis
27. Through (1, -6); parallel to the line $x + 2y = 6$
28. y-intercept 6; parallel to the line $2x + 3y + 4 = 0$
29. Through (-1, 2); parallel to the line $x = 5$
30. Through (2, 6); perpendicular to the line $y = 1$
31. Through (-1, -2); perpendicular to the line $2x + 5y + 8 = 0$
32. Through $(\frac{1}{2}, -\frac{2}{3})$; perpendicular to the line $4x - 8y = 1$
33. Through (1, 7); parallel to the line passing through (2, 5) and (-2, 1)
34. Through (-2, -11); perpendicular to the line passing through (1, 1) and (5, -1)
35. (a) Sketch the line with slope $\frac{3}{2}$ that passes through the point (-2, 1).
(b) Find an equation for this line.
36. (a) Sketch the line with slope -2 that passes through the point (4, -1).
(b) Find an equation for this line.

37-40 ■ Use a graphing device to graph the given family of lines in the same viewing rectangle. What do the lines have in common?

37. $y = -2x + b$ for $b = 0, \pm 1, \pm 3, \pm 6$
38. $y = mx - 3$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$
39. $y = m(x - 3)$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$
40. $y = 2 + m(x + 3)$ for $m = 0, \pm 0.5, \pm 1, \pm 2, \pm 6$

41-52 ■ Find the slope and y-intercept of the line and draw its graph.

- | | |
|---|-------------------------|
| 41. $x + y = 3$ | 42. $3x - 2y = 12$ |
| 43. $x + 3y = 0$ | 44. $2x - 5y = 0$ |
| 45. $\frac{1}{2}x - \frac{1}{3}y + 1 = 0$ | 46. $-3x - 5y + 30 = 0$ |
| 47. $y = 4$ | 48. $4y + 8 = 0$ |
| 49. $3x - 4y = 12$ | 50. $x = -5$ |
| 51. $3x + 4y - 1 = 0$ | 52. $4x + 5y = 10$ |

53. Use slopes to show that $A(1, 1)$, $B(7, 4)$, $C(5, 10)$, and $D(-1, 7)$ are vertices of a parallelogram.
54. Use slopes to show that $A(-3, -1)$, $B(3, 3)$, and $C(-9, 8)$ are vertices of a right triangle.
55. Use slopes to show that $A(1, 1)$, $B(11, 3)$, $C(10, 8)$, and $D(0, 6)$ are vertices of a rectangle.
56. Use slopes to determine whether the given points are collinear (lie on a line).
(a) (1, 1), (3, 9), (6, 21)
(b) (-1, 3), (1, 7), (4, 15)
57. Find an equation of the perpendicular bisector of the line segment joining the points $A(1, 4)$ and $B(7, -2)$.
58. Find the area of the triangle formed by the coordinate axes and the line

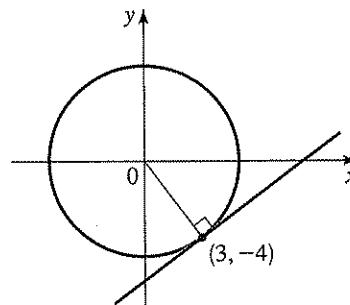
$$2y + 3x - 6 = 0$$

59. (a) Show that if the x- and y-intercepts of a line are nonzero numbers a and b , then the equation of the line can be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

This is called the **two-intercept form** of the equation of a line.

- (b) Use part (a) to find an equation of the line whose x-intercept is 6 and whose y-intercept is -8.
60. (a) Find an equation for the line tangent to the circle $x^2 + y^2 = 25$ at the point (3, -4). (See the figure.)
(b) At what other point on the circle will a tangent line be parallel to the tangent line in part (a)?



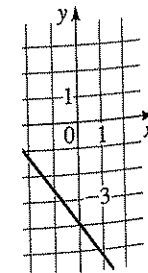
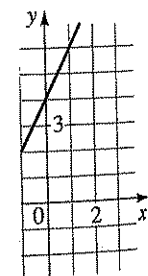
0, so

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0, $\frac{1}{2}$, 2, and -1.

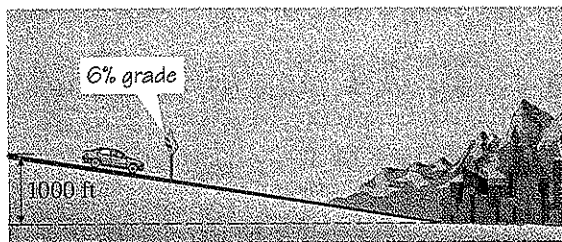
$\frac{1}{2}$, $-\frac{1}{3}$, and 3.

ph is sketched.



Applications

- 61. Grade of a Road** West of Albuquerque, New Mexico, Route 40 eastbound is straight and makes a steep descent toward the city. The highway has a 6% grade, which means that its slope is $-\frac{6}{100}$. Driving on this road you notice from elevation signs that you have descended a distance of 1000 ft. What is the change in your horizontal distance?



- 62. Global Warming** Some scientists believe that the average surface temperature of the world has been rising steadily. The average surface temperature is given by

$$T = 0.02t + 8.50$$

where T is temperature in $^{\circ}\text{C}$ and t is years since 1900.

- (a) What do the slope and T -intercept represent?
 (b) Use the equation to predict the average global surface temperature in 2100.
- 63. Drug Dosages** If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a , pharmacists use the equation

$$c = 0.0417D(a + 1)$$

Suppose the dosage for an adult is 200 mg.

- (a) Find the slope. What does it represent?
 (b) What is the dosage for a newborn?
- 64. Flea Market** The manager of a weekend flea market knows from past experience that if she charges x dollars for a rental space at the flea market, then the number y of spaces she can rent is given by the equation $y = 200 - 4x$.
- (a) Sketch a graph of this linear equation. (Remember that the rental charge per space and the number of spaces rented must both be nonnegative quantities.)
 (b) What do the slope, the y -intercept, and the x -intercept of the graph represent?
- 65. Production Cost** A small-appliance manufacturer finds that if he produces x toaster ovens in a month his production cost is given by the equation

$$y = 6x + 3000$$

(where y is measured in dollars).

- (a) Sketch a graph of this linear equation.
 (b) What do the slope and y -intercept of the graph represent?

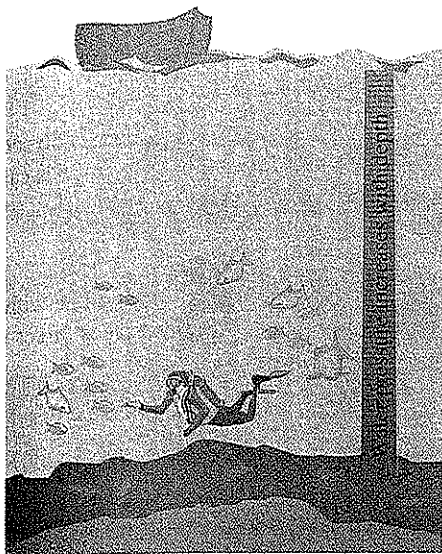
- 66. Temperature Scales** The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by the equation $F = \frac{9}{5}C + 32$.

- (a) Complete the table to compare the two scales at the given values.
 (b) Find the temperature at which the scales agree. [Hint: Suppose that a is the temperature at which the scales agree. Set $F = a$ and $C = a$. Then solve for a .]

C	F
-30°	
-20°	
-10°	
0°	
	50°
	68°
	86°

- 67. Crickets and Temperature** Biologists have observed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 120 chirps per minute at 70°F and 168 chirps per minute at 80°F .
- (a) Find the linear equation that relates the temperature t and the number of chirps per minute n .
 (b) If the crickets are chirping at 150 chirps per minute, estimate the temperature.
- 68. Depreciation** A small business buys a computer for \$4000. After 4 years the value of the computer is expected to be \$200. For accounting purposes, the business uses *linear depreciation* to assess the value of the computer at a given time. This means that if V is the value of the computer at time t , then a linear equation is used to relate V and t .
- (a) Find a linear equation that relates V and t .
 (b) Sketch a graph of this linear equation.
 (c) What do the slope and V -intercept of the graph represent?
 (d) Find the depreciated value of the computer 3 years from the date of purchase.
- 69. Pressure and Depth** At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2 . Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.
- (a) Find an equation for the relationship between pressure and depth below the ocean surface.
 (b) Sketch a graph of this linear equation.
 (c) What do the slope and y -intercept of the graph represent?

- (d) At what depth is the pressure 100 lb/in²?



- 70. Distance, Speed, and Time** Jason and Debbie leave Detroit at 2:00 P.M. and drive at a constant speed, traveling west on I-90. They pass Ann Arbor, 40 mi from Detroit, at 2:50 P.M.
- Express the distance traveled in terms of the time elapsed.
 - Draw the graph of the equation in part (a).
 - What is the slope of this line? What does it represent?
- 71. Cost of Driving** The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May her driving cost was \$380 for 480 mi and in June her cost was \$460 for 800 mi. Assume that there is a linear

relationship between the monthly cost C of driving a car and the distance driven d .

- Find a linear equation that relates C and d .
 - Use part (a) to predict the cost of driving 1500 mi per month.
 - Draw the graph of the linear equation. What does the slope of the line represent?
 - What does the y -intercept of the graph represent?
 - Why is a linear relationship a suitable model for this situation?
- 72. Manufacturing Cost** The manager of a furniture factory finds that it costs \$2200 to manufacture 100 chairs in one day and \$4800 to produce 300 chairs in one day.
- Assuming that the relationship between cost and the number of chairs produced is linear, find an equation that expresses this relationship. Then graph the equation.
 - What is the slope of the line in part (a), and what does it represent?
 - What is the y -intercept of this line, and what does it represent?

Discovery • Discussion

- 73. What Does the Slope Mean?** Suppose that the graph of the outdoor temperature over a certain period of time is a line. How is the weather changing if the slope of the line is positive? If it's negative? If it's zero?
- 74. Collinear Points** Suppose you are given the coordinates of three points in the plane, and you want to see whether they lie on the same line. How can you do this using slopes? Using the Distance Formula? Can you think of another method?

1.11

Modeling Variation

Mathematical models are discussed in more detail in *Focus on Modeling*, which begins on page 239.

When scientists talk about a mathematical model for a real-world phenomenon, they often mean an equation that describes the relationship between two quantities. For instance, the model may describe how the population of an animal species varies with time or how the pressure of a gas varies as its temperature changes. In this section we study a kind of modeling called *variation*.

Direct Variation

Two types of mathematical models occur so often that they are given special names. The first is called *direct variation* and occurs when one quantity is a constant multiple of the other, so we use an equation of the form $y = kx$ to model this dependence.